The Bat-Ball Collision

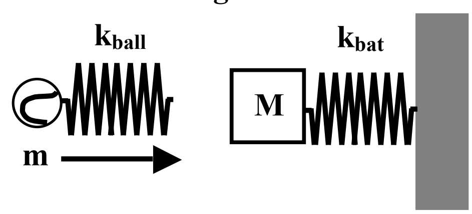
If you throw a baseball against a wall, it bounces back. Yet it has less energy after the bounce than before. In fact, the post-bounce energy is approximately 25% of the pre-bounce energy. The collision is said to be **partially elastic**. The ball's kinetic energy (KE), or energy of motion, is transformed into other forms of energy. For any collision, the **coefficient of restitution** (COR) is the ratio of the post-collision to pre-collision speed of the ball. **Figure 1** shows the formula relating KE to the speed (v) and the mass (m) of an object. It follows from this formula that the COR is about 0.5 for a collision in which approximately 25% of the kinetic energy is maintained by the colliding object. A higher COR would be consistent with a larger post-collision speed.

Figure 1

$$KE = \frac{1}{2} \bullet m \bullet v^2$$

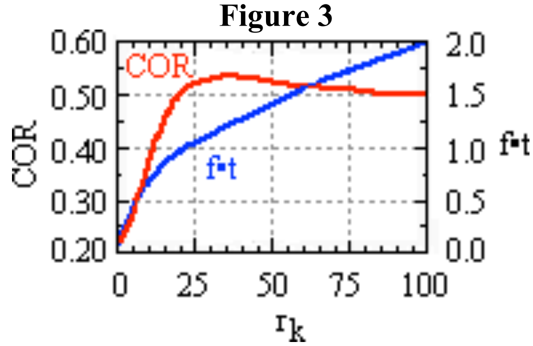
A collision of a baseball with a bat is slightly more elastic than the ball-wall collision. This greater elasticity is due to the flexibility of the bat - an effect referred to as the *trampoline effect*. A simplistic model of the collision represents the ball and bat as compressible springs that are colliding with each other as shown in **Figure 2**. Compressible objects are often described by a **spring constant (k)**. Easily compressed springs have a low spring constant and act like *springy* trampolines. A rigid, difficult-to-compress spring has a larger spring constant. When modeling a bat-ball collision, $\mathbf{k_{bat}}$ and $\mathbf{k_{ball}}$ are used as the spring constants of the bat and the ball.

Figure 2



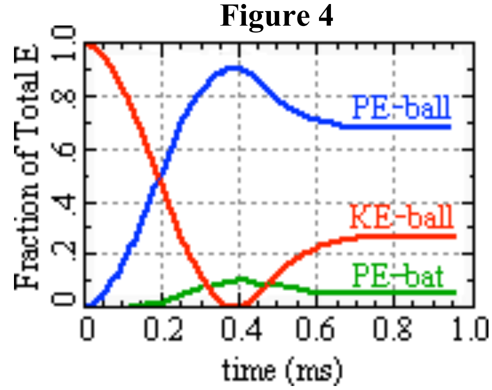
Computer modeling of the bat-ball collision indicates that the ratio of the spring constants ($r_K = k_{bat}/k_{ball}$) seems to be of critical importance to the resulting COR of the collision. **Figure 3** shows the COR as a function of the spring constant ratio. Another variable that is of importance to an effective ball-bat collision is the product **f•t**. This product varies with the ratio of spring constants as shown in **Figure 3**. The **f** is the frequency with which the bat spring naturally vibrates and **t** is the collision time for the collision. A larger f•t value indicates that more of the bat's vibration is transferred back to the ball to enhance the *trampoline effect*.

Figure 3



The ball-bat collision lasts for less than a millisecond (ms). During this time, a portion of the original KE of the ball is transformed into other forms of energy as shown in **Figure 4**. Some of this energy is transformed back into KE once the collision is over. But the majority of the energy exists as internal energy in the ball (PE-ball) and in the bat (PE-bat). This energy is the energy associated with the vibrations of the molecules of the bat and the ball. Like springs, these molecules continue to vibrate long after the contact between the bat and the ball has ceased.

Figure 4



Data in this passage is based on a study by Alan Nathan and Loyd Smith titled "The Physics of the Trampoline Effect in Baseball and Softball Bats ". The study can be found at:

http://webusers.npl.illinois.edu/~a-nathan/pob/trampoline-v6.pdf

Questions:

1. Suppose the speed of an object is doubled. According to the equation in **Figure 1**, how would this doubling of speed affect the kinetic energy?
   a. The kinetic energy would increase by a factor of two.
   b. The kinetic energy would increase by a factor of four.
   c. The kinetic energy would be greater by a factor of 1.41.
   d. The kinetic energy would be greater by a factor of 20.

2. What would be the COR for a collision in which 16% of the original kinetic energy is maintained?
   a. 0.0256 b. 0.04
   c. 0.256 d. 0.40

3. What is the relationship between the COR for a collision and the percent of the original kinetic energy (KE) that is transformed to other forms?
   a. The COR is least in collisions in which the most KE is transformed.
   b. The COR is greatest in collisions in which the most KE is transformed.
   c. The COR is not related to the amount of KE being transformed by the colliding objects.
   d. The COR depends on the amount of KE transformed but the relationship is too complex to predict the effect.

4. Suppose that the ball-wall collision was modeled by the spring model depicted in **Figure 2**. Use **Figure 3** to approximate the value of r_k for the collision.
   a. 20 b. 40
   c. 50 d. 150

5. A bat-ball collision is most elastic when _____.
   a. the COR of the collision is small
   b. there is very little trampoline effect
   c. the ball bounces off the bat at a high speed
   d. the pre-bounce KE is much greater than the post-bounce KE

6. Physicists have modeled the bat-ball collision using a compressible spring model. Which of the following statements describes what the model predicts about the most effective collisions?
   a. The ball and bat must be as flexible as possible.
   b. The bat is more easily compressed than the ball.
   c. The ball is more easily compressed than the bat.
   d. The ball and bat must have spring constants that are equal to one another.

7. According to **Figure 3**, which of the following is true of collisions as the r_k values increase past 35 to higher and higher values?
   a. The **f•t** product remains constant.
   b. The COR value continuously increases.
   c. The bat begins to vibrate at greater energy levels.
   d. The energy dissipated to vibrations and heat within the ball tends to level out.

8. According to **Figure 3**, ball-bat collisions with the greatest COR occur when _____.
a. the k_{ball} is 20/100th the value of k_{bat}.
b. the k_{bat} is 20 times the value of k_{ball}.
c. the k_{bat} is 35 times the value of k_{ball}.
d. the k_{bat} is 1.7 times the value of k_{ball}.

9. According to **Figure 3**, what affect does increasing the ratio of spring constants (r_k) have upon the COR?
a. The COR increases.
b. The COR decreases.
c. The COR does not change.
d. The COR first increases and then decreases.

10. A bat has a spring constant that is 75 times greater than the spring constant of the ball. According to **Figure 3**, what is the COR and f•t value?
a. The COR value is approximately 0.50 and the f•t value is approximately 1.5.
b. The COR value is approximately 0.50 and the f•t value is approximately 1.7.
c. The COR value is approximately 0.55 and the f•t value is approximately 1.5.
d. The COR value is approximately 0.55 and the f•t value is approximately 1.7.

11. Which one of the following statements accurately describes what happens during a collision between a baseball and a bat?
a. The potential energy of the ball (E-ball) is a maximum when the kinetic energy of the ball (KE-ball) is a minimum.
b. The potential energy of the ball (PE-ball) continuously increases during the collision.
c. The kinetic energy of the ball (KE-ball) continuously decreases during the collision.
d. The energy of the bat (E-bat) is a maximum when the potential energy of the ball (PE-ball) is a maximum.

12. According to **Figure 4**, the ball has lost all its kinetic energy _____ after the start of the collision?
a. immediately
b. 0.38 milliseconds
c. 0.67 milliseconds
d. 1.0 milliseconds

13. As illustrated in **Figure 4**, the original energy of the ball becomes transformed into other energy forms shortly after the contact between bat and ball begins. At a time of 0.50 milliseconds after contact begins, ______ % of the energy is in the ball's vibration, ______ % of the energy is in the bat's vibrations, and _____ % of the energy is in the form of the ball's kinetic energy.
a. 70, 5, 25
b. 70, 25, 5
c. 75, 10, 15
d. 75, 15, 10