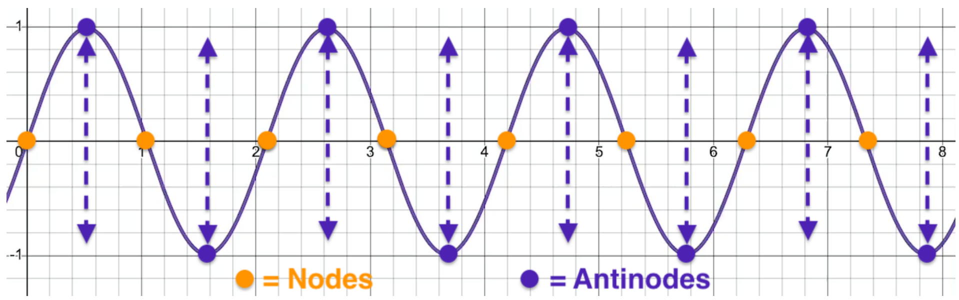


Standing Waves and Harmonics
Lesson Notes

Learning Outcomes
- How do you draw the standing wave patterns for the various harmonics?
- How are the frequencies and wavelengths for the various harmonics related?

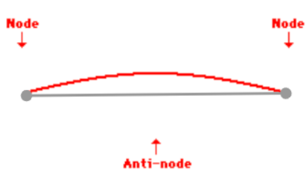
Standing Wave Patterns
A **standing wave** is a pattern resulting from the interference of two waves that have just the right frequency to cause points along the medium to appear to be standing still and other points to be vibrating wildly.



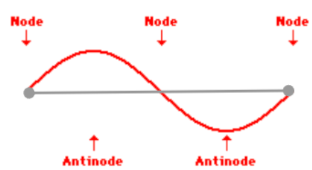
Standing waves consist of …
- **Nodes**: points of **no des**placement; destructive interference
- **Antinodes**: maximum +/- displacement; constructive interference

First Three Harmonics
When vibrated at just the right frequency, a rope will vibrate as a standing wave. Each frequency, known as a **harmonic frequency**, will result in its own unique standing wave pattern or **harmonic**.

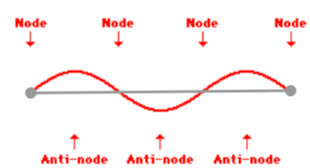
First Harmonic



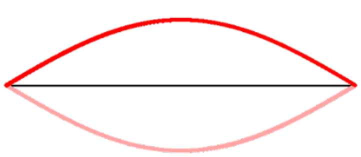
Second Harmonic

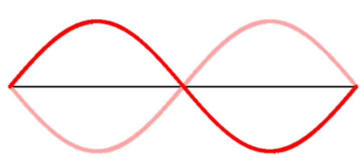


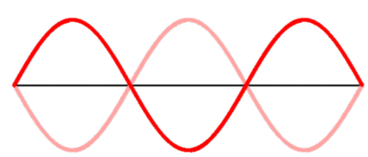
Third Harmonic



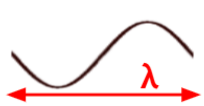
Standing wave diagrams show the position of the antinodes 2X/cycle.



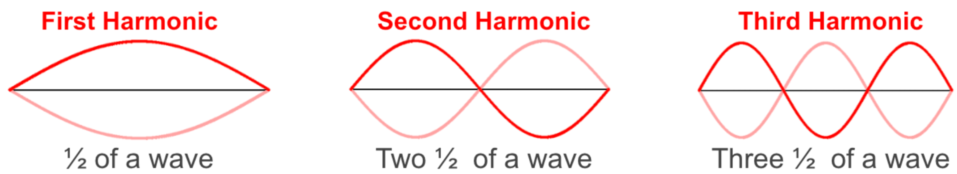




Wavelength-Frequency Relationships

This is a wave: 

So this is ½ of a wave: 



The 1[st] harmonic has a λ that is 2X the λ of the 2[nd] harmonic. ➡ $\lambda_1 = 2 \cdot \lambda_2$

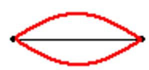
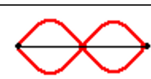
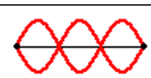
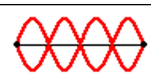
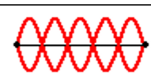
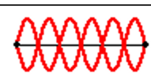
The 1[st] harmonic has a λ that is 3X the λ of the 3[rd] harmonic. ➡ $\lambda_1 = 3 \cdot \lambda_3$

Since v = f•λ and since v is the same for each harmonic, so as λ↓, f↑.

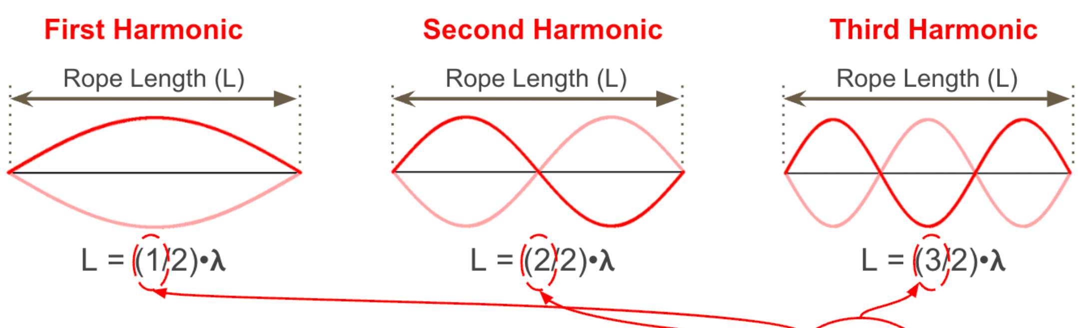
The 1[st] harmonic has an f that is ½ X the f of the 2[nd] harmonic. ➡ $f_1 = \frac{1}{2} \cdot f_2$

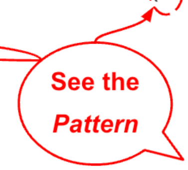
The 1[st] harmonic has an f that is ⅓ X the f of the 3[rd] harmonic. ➡ $f_1 = \frac{1}{3} \cdot f_3$

Wave Patterns and Numerical Patterns

Harmonic	Pattern	# of Nodes	# of Antinodes	λ	f	Examples	
						λ (m)	f (Hz)
1[st]		2	1	λ_1	f_1	1.20	50
2[nd]		3	2	$\lambda_1/2$	$2 \cdot f_1$	0.60	100
3[rd]		4	3	$\lambda_1/3$	$3 \cdot f_1$	0.40	150
4[th]		5	4	$\lambda_1/4$	$4 \cdot f_1$	0.30	200
5[th]		6	5	$\lambda_1/5$	$5 \cdot f_1$	0.24	250
6[th]		7	6	$\lambda_1/6$	$6 \cdot f_1$	0.20	300
n[th]	--	n+1	n	λ_1/n	$n \cdot f_1$	1.20/n	50•n

Length-Wavelength Relationships





In general, **$L = (n/2) \cdot \lambda$** where **n** = harmonic #

L = length of rope

General equation for determining the **λ** from the length (**L**) is:

$\lambda = (2/n) \cdot L$