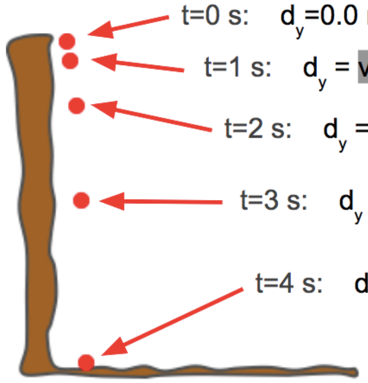
x- and y-Displacement of a Projectiles

Lesson Notes

Vertical Displacement of a Projectile

Kinematic Equation: $d = v_o \cdot t + \frac{1}{2} \cdot a \cdot t^2 \Rightarrow d_y = \cancel{v_{oy} \cdot t} + \frac{1}{2} \cdot a_y \cdot t^2$



t=0 s: $d_y = 0.0$ m

t=1 s: $d_y = v_{oy} \cdot t + \frac{1}{2} \cdot a_y \cdot t^2 = \frac{1}{2} \cdot (-9.8 \text{ m/s}^2) \cdot (1 \text{ s})^2 = -4.9$ m

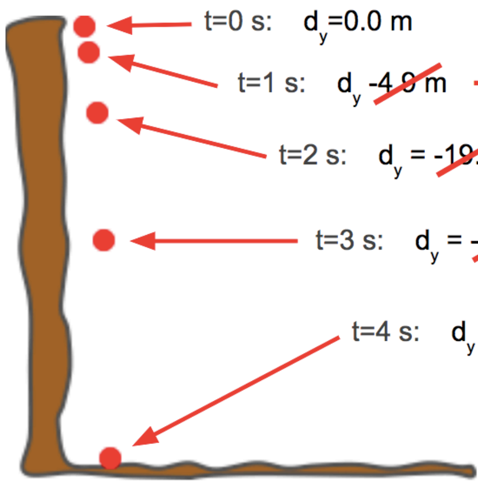
t=2 s: $d_y = v_{oy} \cdot t + \frac{1}{2} \cdot a_y \cdot t^2 = \frac{1}{2} \cdot (-9.8 \text{ m/s}^2) \cdot (2 \text{ s})^2 = -19.6$ m

t=3 s: $d_y = \frac{1}{2} \cdot (-9.8 \text{ m/s}^2) \cdot (3 \text{ s})^2 = -44.1$ m

t=4 s: $d_y = \frac{1}{2} \cdot (-9.8 \text{ m/s}^2) \cdot (4 \text{ s})^2 = -78.4$ m

The - means down.

For some quick, back-of-the-envelope calculations, a value of 10 m/s/s is often used for the value of the vertical acceleration.



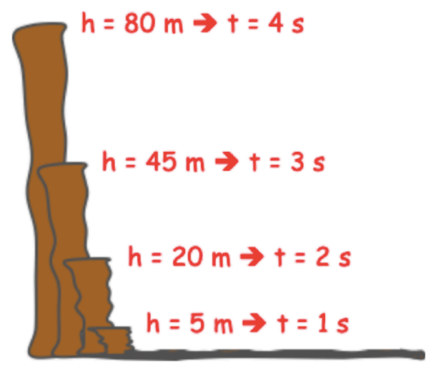
t=0 s: $d_y = 0.0$ m

t=1 s: d_y ~~-4.9 m~~ **- 5 m**

t=2 s: d_y = ~~-19.6 m~~ **- 20 m**

t=3 s: d_y = ~~-44.1 m~~ **- 45 m**

t=4 s: d_y = ~~-78.4 m~~ **- 80 m**



Horizontal Displacement of a Projectile

The horizontal displacement (d_x) depends upon the original horizontal velocity (v_{ox}) and the time (t) of fall.

Kinematic Equation
$d = v_o \cdot t + \frac{1}{2} \cdot a \cdot t^2$
$d_x = v_{ox} \cdot t$ [redacted]

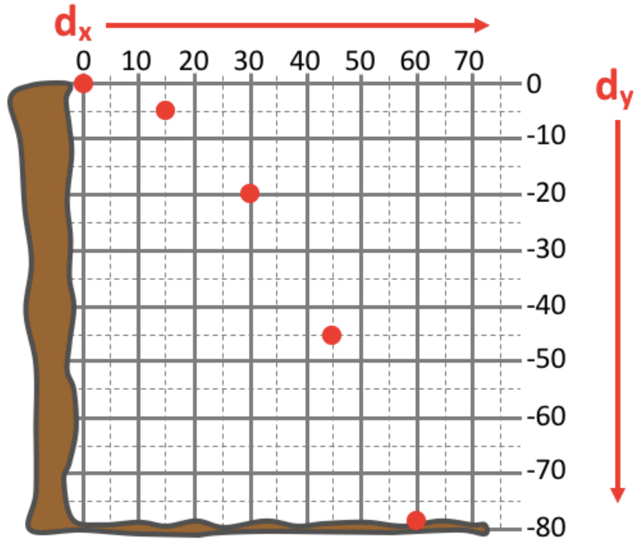
Consider a ball launched horizontally at 15 m/s from the top of an 80-m high cliff.

t (s)	d_x (m)	d_y (m)
0	0	0
1	15	-5
2	30	-20
3	45	-45
4	60	-80

Trajectory Plot

A ball is launched horizontally at 15 m/s from the top of an 80-m high cliff.

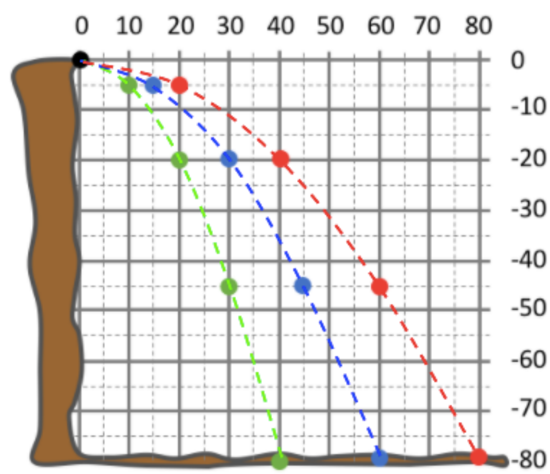
The trajectory of a projectile is parabolic in shape because of the vertical acceleration ($d_y \propto t^2$) and the constant horizontal velocity ($d_x \propto t$).



d_x Depends on v_{ox}

Consider three horizontal launch velocities for a projectile launched from the top of an 80-m high cliff: **10 m/s**, **15 m/s** and **20 m/s.**

The time to fall - 4 seconds - is not affected by the v_{ox} value. The horizontal displacement (d_x) is affected by the v_{ox} value.



Angle-Launched Trajectory

Imagine a ball launched at an angle above the horizontal with v_{ox} of 12 m/s and v_{oy} of 20 m/s.

t (s)	d_x (m)	$v_{oy} \cdot t$ (m)	$\frac{1}{2} \cdot (-10) \cdot t^2$ (m)	dy (m)
0	0	0	0	0
1	12	20	-5	+15
2	24	40	-20	+20
3	36	60	-45	+15
4	48	80	-80	0
5	60	100	-125	-25
6	72	120	-180	-60

$$d_y = v_{oy} \cdot t + \frac{1}{2} \cdot (-10) \cdot t^2$$

Gravity-free Path

Gravity's affect