Mathematics of a Projectiles

Lesson Notes

Kinematic Equations for Projectile Motion

Kinematics equations apply to objects moving along straight lines with a uniform acceleration between an initial and a final state.

$d = v_o \cdot t + ½ \cdot a \cdot t^2$

$v_f = v_o + a \cdot t$

$v_f^2 = v_o^2 + 2 \cdot a \cdot d$

$d = [(v_o + v_f)/2] \cdot t$

d = displacement
a = acceleration
t = time
v_o = original velocity
v_f = final velocity

For horizontal motion (x)

$d_x = v_{ox} \cdot t$ [redacted]

$v_{fx} = v_{ox}$ [redacted]

$v_{fx}^2 = v_{ox}^2$ [redacted]

$d_x = [(v_{ox} + v_{fx})/2] \cdot t$

Know values of 3 variables; calculate the 4th value.

For vertical motion (y)

$d_y = v_{oy} \cdot t - 4.9 \cdot t^2$

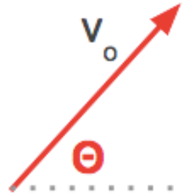
$v_{fy} = v_{oy} - 9.8 \cdot t$

$v_{fy}^2 = v_{oy}^2 - 19.6 \cdot d_y$

$d_y = [(v_{oy} + v_{fy})/2] \cdot t$

The Original Velocity

Most projectile problems provide information about the original velocity (v_o) and the angle (θ). You must begin by resolving v_o into x- and y-components (v_{ox} and v_{oy}).



$v_{ox} = v_o \cdot \cos \Theta$

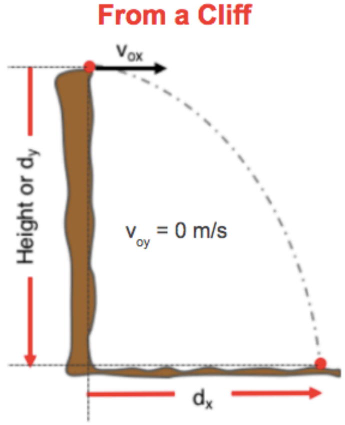
$v_{oy} = v_o \cdot \sin \Theta$

where Θ is the launch angle measured with the ground.

NEVER use values of v_o and Θ in kinematic equations.

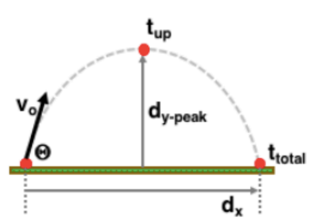
Three Basic Problem Types

#1 Horizontally Launched Projectiles From a Cliff



Solve for d_x, v_{ox}, or d_y

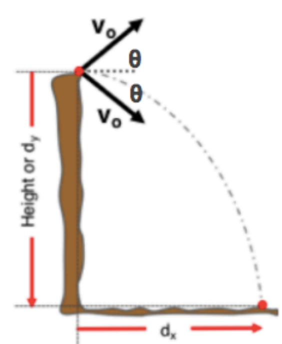
#2 Angle Launched Projectiles From Ground



Given v_o and θ

Solve for t_{up}, t_{total}, d_x, and $d_{y\text{-peak}}$

#3 Angle Launched Projectiles From a Cliff



Projectile Velocity and Acceleration

A projectile does not accelerate horizontally.

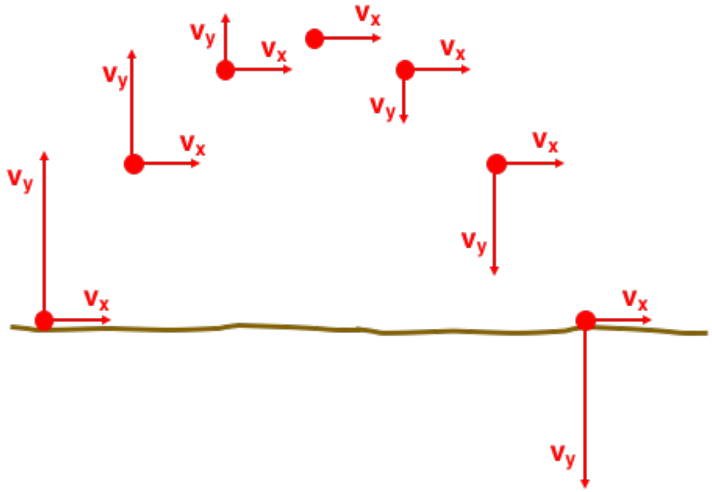
A projectile accelerates vertically at -9.8 m/s/s (the - means down). So v_{fy} for any time t can be calculated …

$$\mathbf{v_{fy} = v_{oy} - 9.8 \cdot t}$$

The final y-velocity (v_{fy}) is equal and opposite the initial y-velocity (v_{oy}):

$\mathbf{v_{fy} = -v_{oy}}$

This fact can be used in many calculations.



At the highest point ("peak"), the y-velocity is 0 m/s. This fact can be used in many calculations.

Angle-Launched Projectiles - Time

For angle-launched projectiles, there's a mathematical relationship between the original y-velocity (v_{oy}), the time to rise up to the peak (t_{up}) and the total time in the air (t_{total}).

From $v_{fy} = v_{oy} - 9.8 \cdot t$, you can derive ...

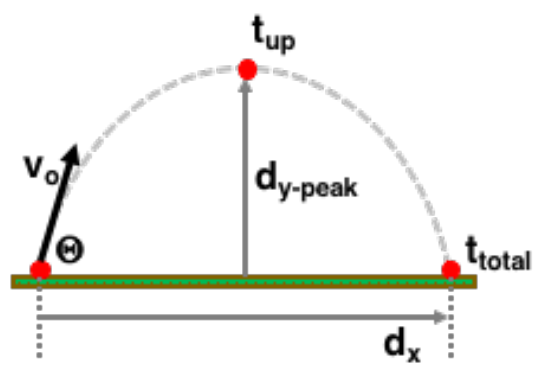
$$0 \text{ m/s} = v_{oy} - (9.8\text{m/s/s}) \cdot t_{up}$$

$$\mathbf{t_{up} = v_{oy} / 9.8}$$

And since $t_{up} = t_{down}$, $\mathbf{t_{total} = 2 \cdot t_{up}}$.

Angle-Launched Projectiles - Displacement

The $\mathbf{t_{up}}$ value refers to the time to travel through ½ the trajectory … to the highest point. The $\mathbf{t_{total}}$ value refers to the time to travel up and down - the full trajectory. Knowing $\mathbf{t_{up} = v_{oy} / 9.8}$, you can calculate the height of the object at the peak ($d_{y\text{-}peak}$) and the total x-displacement (d_x) upon landing on the ground.



Height at Peak

Use $d_y = [\,(v_{oy} + v_{fy})/2\,] \cdot t$

where $v_{fy} = 0$ m/s.

$\mathbf{d_{y\text{-}peak} = v_{oy}/2 \cdot t_{up}}$

Horizontal Displacement

Use $d_x = v_{ox} \cdot t$

where t is total time (t_{total}).

$\mathbf{d_x = v_{ox} \cdot t_{total}}$