Modified Atwood's Machines

Lesson Notes

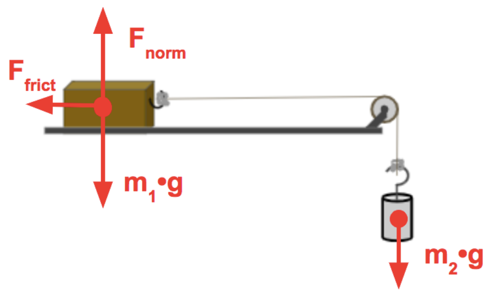
Learning Outcomes

- How do you use a free-body diagram and Newton's second law to analyze and solve a modified Atwood's Machine problem?

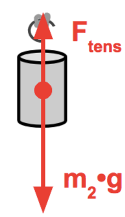
The Bsic Approach

A modified Atwood's Machine problem can be analyzed in two steps.

- **A System Analysis**
  Relate the acceleration to the mass values.

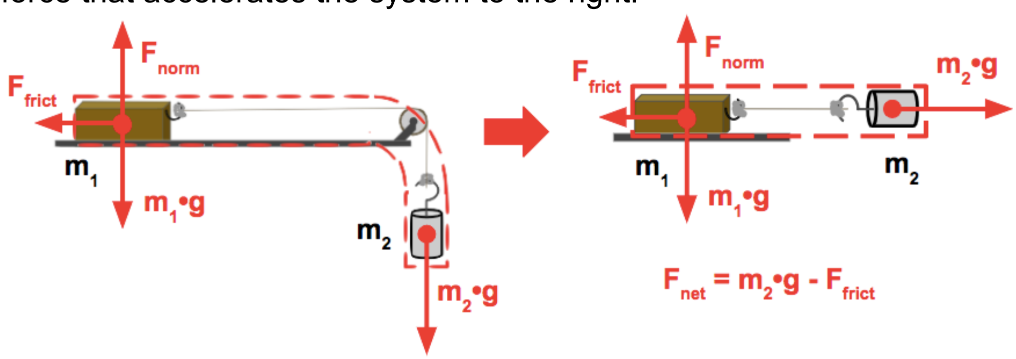


- **An Individual Object Analysis**
  Relate the tension in the string to the acceleration and the mass values.



Straightening Out the System

A convenient strategy involves picturing the gravity force on the hanging mass to be a horizontal force that accelerates the system to the right.



$F_{net} = m_2 \cdot g - F_{frict}$

Example 1:

A 0.150-kg hanging mass (m_2) is attached by a string to a 0.500-kg cart (m_1) at rest on a friction-free table. Calculate the acceleration of the cart and the tension in the string.

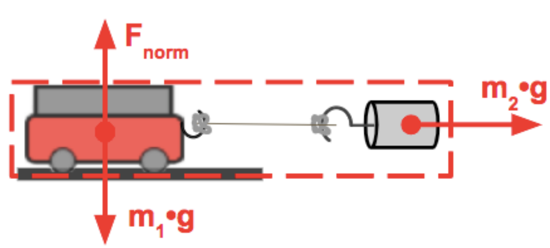
Step 1: System Analysis

$m_{total} = 0.650$ kg

$F_{net} = m_2 \cdot g$

$F_{net} = (0.150\text{ kg}) \cdot (9.8\text{ N/kg})$

$F_{net} = 1.47$ N



$a = F_{net} / m_{total} = (1.47\text{ N})/(0.650\text{ kg}) =$ **2.26 m/s/s**

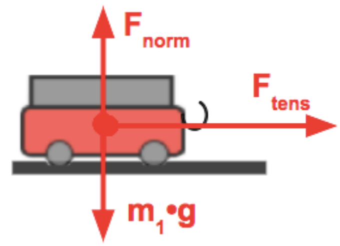
Step 2: Individual Object Analysis

Consider $\mathbf{m_1}$: $m_1 = 0.500$ kg

$F_{net} = m_1 \cdot a$

$F_{net} = (0.500 \text{ kg}) \cdot (2.26 \text{ m/s/s})$

$F_{net} = 1.13$ N



The vertical forces balance. The tension force is the one unbalanced force that causes the acceleration. It equals F_{net}.

$$\mathbf{F_{tens} = 1.13 \text{ N}}$$

Example 2:

A 0.250-kg hanging mass (m_2) is attached by a string to a 0.500-kg block (m_1) at rest on a table. The coefficient of friction (μ) is 0.215. Calculate the acceleration of the cart and the tension in the string.

Step 1: System Analysis

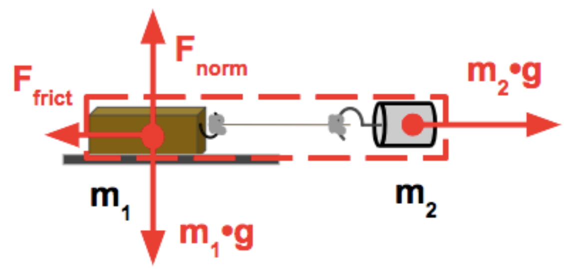
$m_{total} = 0.750$ kg

$m_2 \cdot g = (0.250 \text{ kg}) \cdot (9.8 \text{ N/kg}) = 2.45$ N

$F_{frict} = \mu \cdot F_{norm} = \mu \cdot m_1 \cdot g$

$F_{frict} = (0.215) \cdot (0.500 \text{ kg}) \cdot (9.8 \text{ N/kg})$

$F_{frict} = 1.05$ N (1.0535 N)



$F_{net} = m_2 \cdot g - F_{frict} = 2.45 \text{ N} - 1.05 \text{ N} = \mathbf{1.40 \text{ N}}$

$a = F_{net} / m = (1.40 \text{ N})/(0.750 \text{ kg}) = \mathbf{1.86 \text{ m/s/s}}$

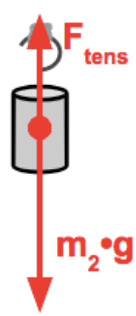
Step 2: Individual Object Analysis

Consider $\mathbf{m_2}$: $m_2 = 0.250$ kg

$m_2 \cdot g = (0.250 \text{ kg}) \cdot (9.8 \text{ N/kg}) = 2.45$ N

$F_{net} = m_2 \cdot a = (0.250 \text{ kg}) \cdot (1.86 \text{ m/s/s})$

$F_{net} = 0.466$ N (0.4655 N)



$F_{net} = m_2 \cdot g - F_{tens}$

$0.466 \text{ N} = 2.45 \text{ N} - F_{tens}$

$F_{tens} = 2.45 \text{ N} - 0.466 \text{ N}$

$F_{tens} = \mathbf{1.98 \text{ N}}$