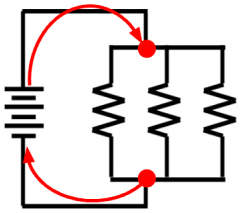
Parallel Circuit Relationships
Lesson Notes

Learning Outcomes
- What are the important mathematical patterns and relationships associated with parallel circuits?
- How do you use these relationships?

What is a Parallel Circuit?

Parallel Circuit



Branches!!!

Multiple pathways.

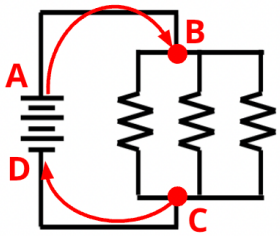
Charge passes through just one of the resistors.

For Parallel Circuits:

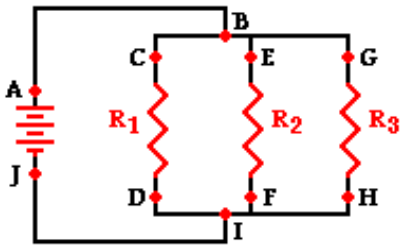
As the number of resistors increases …
… total resistance decreases
… current increases, and
Removing a bulb from its socket has no effect on the other bulbs.

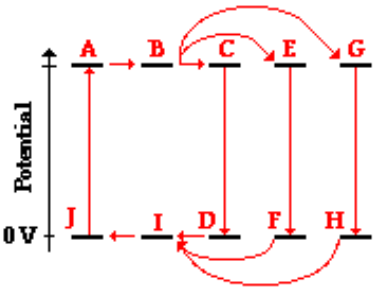
Voltage Drops Across the Branches

- Charge traversing *the loop* of a parallel circuit will only pass through one branch before returning to the battery.
- There is a voltage gain in the battery and a voltage drop in the branches. These must be equal.



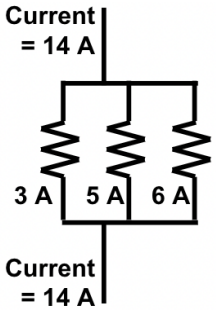
$$\Delta V_{battery} = \Delta V_1 = \Delta V_2 = \Delta V_3 = \ldots$$



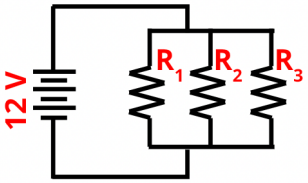


Current

- The current outside the branches is equal to the sum of the branch currents. $I_{battery} = I_1 + I_2 + I_3 + \ldots$
- Current can be calculated using: $I_1 = \frac{\Delta V}{R_1}$ $I_2 = \frac{\Delta V}{R_2}$ $I_3 = \frac{\Delta V}{R_3}$



Example: Three resistors are connected to a 12 V battery. $R_1 = 2\ \Omega$, $R_2 = 6\ \Omega$, and $R_3 = 4\ \Omega$. Determine I_1, I_2, and I_3.



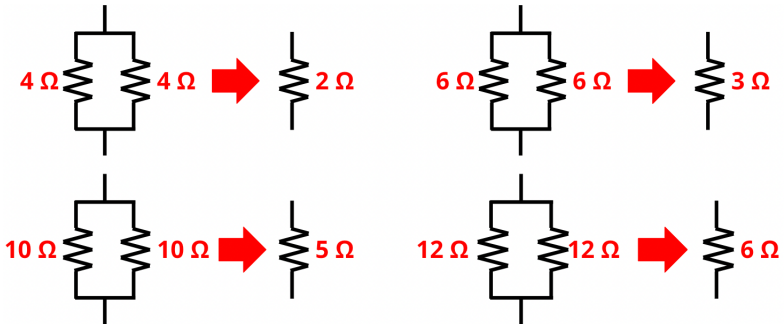
$I_1 = (12\ V)/(2\ \Omega) = \mathbf{6\ A}$

$I_2 = (12\ V)/(6\ \Omega) = \mathbf{2\ A}$

$I_3 = (12\ V)/(4\ \Omega) = \mathbf{3\ A}$

Equivalent Resistance

The **equivalent resistance** (R_{eq}) of a multiple-resistor circuit is the amount of resistance a single resistor must have to match the effect of the collection of resistors.



For parallel circuits, the equivalent resistance is calculated as

$$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$$

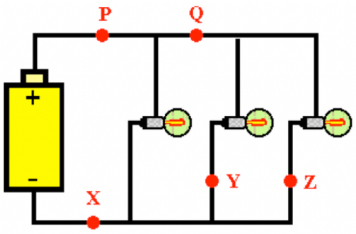
R_1 (Ω)	R_2 (Ω)	R_3 (Ω)	$1/R_1 + 1/R_2 + 1/R_3$	R_{eq} (Ω)
4.0	4.0	–	1/(4.0) + 1/(4.0) = 0.50	2.0
12	12	–	1/(12) + 1/(12) = 0.1666…	6.0
12	12	12	1/(12) + 1/(12) + 1/(12) = 0.25	4.0
2.0	3.0	4.0	1/(2.0) + 1/(3.0) + 1/(4.0) = 1.083	0.92
4.5	6.6	5.1	1/(4.5) + 1/(6.6) + 1/(5.1) = 0.5698	1.75

The R_{eq} value is always smaller than the smallest R value. Adding a resistor in a parallel branch always makes the overall resistance less.

Concept Practice

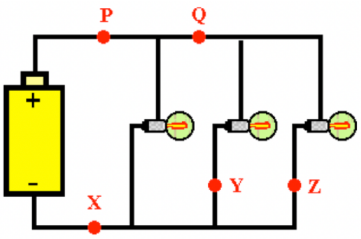
Identify and explain the answers to the following Concept Questions:

Three identical light bulbs are connected to a battery. P, Q, X, Y and Z represent locations along the circuit. Which one of the statements is true?



a. The current at Y is greater than the current at Q.
b. The current at Y is greater than the current at P.
c. The current at Y is greater than the current at Z.
d. The current at P is greater than the current at Q.
e. The current at Q is greater than the current at P.
f. The current is the same at all locations.

Which adjustments could be made to the circuit that would increase the current at X? Identify all that apply.



a. Increase the resistance of one of the bulbs.
b. Increase the resistance of two of the bulbs.
c. Decrease the resistance of two of the bulbs.
d. Increase the voltage of the battery.
e. Decrease the voltage of the battery.
f. Remove one of the bulbs.