Mathematics of Musical Instruments

1. A guitar string has a length of 72.1 cm and a mass of 0.425 grams. If tightened to a tension of 136N, at what speed will vibrations travel through the string?

2. The E5-string of a violin has a mass density of 0.38 g/m and a length from peg to bridge of 33 cm. To what tension must the string be tightened in order to play 660 Hz?

3. A string stretched between fixed posts 250 cm apart vibrates with a fundamental frequency of 100 Hz. What is the speed at which vibrations travel in the string? And what are the frequencies of the second, third and fourth harmonics?

4. Dennis Elbow strings his own tennis rackets. He likes to "play" the cross strings on his racket before completing the basket weave. One of his favorite strings seems to play a fundamental frequency of 720 Hz when plucked. The string has an effective length of 24 cm (between the sides of the frame). Dennis strings his racket to a tension of 260 N. Determine the mass density of his tennis string.

5. A B-string from a guitar is held fixed at both ends and has a vibrating length of 33 cm. It oscillates at its fundamental frequency of 246 Hz. What are the wavelengths on the string? What are the wavelengths of the sound waves which it produces? (Assume a speed of sound in air of 345 m/s.)

6. A piece of steel piano wire is held fixed at both ends under a tension of 113 N. The free length of wire is 89.2 cm and it has a mass of 2.59 grams. What are the frequencies of the first three harmonics which it can play?

7. A narrow tube is 1.00 m long and closed rigidly at one end with a piston at the other. Given that the speed of sound in the air inside the tube is 340 m/s, determine the fundamental frequency of the tube.

8. The auditory canal of the outer ear is closed at one end by the ear drum and open to the surroundings at the other end. Its length is approximately 2.7 cm long. It serves as a closed-end resonator, amplifying certain sounds more than others. Assuming a sound speed of 345 m/s, determine the fundamental frequency of this standing wave cavity.

9. If your auditory canal was effectively 200 times longer (as it is on an elephant), what would be its fundamental frequency?

10. A 140 cm long organ pipe is resonating as a closed-end air column. Determine the fundamental frequency and the first three overtones (i.e., the next three higher harmonics) which it can sound out. Assume a temperature of 20 °C.

11. A C-flute with all its holes covered plays a middle C (262 Hz) as its fundamental. Assuming a room temperature of 20 °C and that the flute serves as an open-end resonator, how long would the flute be from embouchre hole to end.

12. How far from the end of the flute should the hole that must be uncovered be in order to play note D above middle C (f = 294 Hz)? (Use the same temperature as in question #11.)

13. Wind chimes act as open-end air columns. Chime A is 1.7 times longer than chime B. The third harmonic of chime B is 521 Hz. Determine the fundamental frequency of chime A.

14. An instrument acts as an open-end resonator and plays out a third harmonic frequency of 693 Hz. Determine the frequency of the fifth harmonic of an open-end instrument which is one-half the length.

15. A 50-cm wire plays the same fundamental frequency as the third harmonic frequency of a 29-cm closed-end air column (at 20 °C). Determine the fundamental frequency of a second wire with twice the length, one-half the linear density and three times the tension.

16. Two identical piano strings are both tuned to 440 Hz. The tension in one of them is increased by 1.00 %. Both strings are activated so that they sound out with their fundamental frequencies. What will be the resulting beat frequency?