

Satellite Motion

Read from **Lesson 4** of the **Circular and Satellite Motion** chapter at **The Physics Classroom**:

<http://www.physicsclassroom.com/Class/circlesu614b.html>

<http://www.physicsclassroom.com/Class/circles/u614c.html>

MOP Connection: Circular Motion and Gravitation: sublevel 8

1. Consider the rather strange-looking orbiting satellite shown in the diagram at the right. Draw a free-body diagram showing the type and direction of the forces acting upon the satellite.



2. For any satellite, the net force is equal to $m_{\text{sat}} \cdot a$ or $m_{\text{sat}} \cdot \frac{v^2}{R}$. Since this net centripetal force is supplied by the force of gravity, the force of gravity expression can be set equal to the net centripetal force expression:

$$\frac{G \cdot m_{\text{sat}} \cdot M_{\text{earth}}}{R^2} = m_{\text{sat}} \cdot \frac{v^2}{R}$$

Algebraically manipulate this equation in order to derive an expression for the speed (v) of an orbiting satellite. **PSAYW**

3. Use your equation in #2 above to answer the following questions:
 - If the radius of orbit of a satellite is increased, then the orbital speed would _____.
 - If mass of the earth is increased, then the orbital speed would _____.
 - If the radius of the earth is increased, then the orbital speed would _____.
 - If the mass of the satellite is increased, then the orbital speed would _____.
 - If the radius of orbit of a satellite is increased by a factor of 2 (i.e., doubled), then the orbital speed would _____ (increase, decrease) by a factor of _____.
 - If the mass of the earth is increased by a factor of 2 (i.e., doubled), then the orbital speed would _____ (increase, decrease) by a factor of _____.
4. Use the equation in derived #2 to calculate the orbital speed of ($M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg}$)

| Object | Orbital Radius (m) | Orbital Speed (m/s) |
|-----------------------------------|--------------------|---------------------|
| a. ... the moon | 4×10^8 | |
| b. ... a geosynchronous satellite | 4.15×10^7 | |
| c. ... the space shuttle | 6.55×10^6 | |

Circular and Satellite Motion

5. The speed of a satellite is also found from its orbital period (**T**) and the radius of orbit (**R**):

$$v = \frac{2 \cdot \pi \cdot R}{T}$$

Set the expression for orbital speed (**v**) above to the expression for orbital speed from question #2. Algebraically manipulate the equation to obtain an equation relating orbital period (**T**) to the radius and mass of the earth.

Analyze the following trip knowing the concepts and equations utilized in this unit. Insert your answers to the following questions in the table below.

6. Suppose that the man pictured on the front side is orbiting the earth (mass = 5.98×10^{24} kg) at a distance of 310 miles (1600 meters = 1 mile) above the surface of the earth (radius = 4000 miles).
- What acceleration does he experience due to the earth's pull?
 - What tangential velocity must he possess in order that to orbit safely (in m/s)?
 - What is his period (in hours)?
7. Now suppose that the man is orbiting the earth at 22,500 miles above its surface.
- What is the acceleration?
 - What is the tangential velocity (in m/h) at this location?
 - What is his period (in hours)?
8. Finally suppose that the man lands on the moon ($R_{\text{earth-moon}} = 4 \times 10^8$ meters).
- What is the moon's and its inhabitants acceleration (in m/s^2) around the earth?
 - What is the tangential velocity (in m/s) around the earth?
 - What is the moon's period (in days)?

| Object | Radius (m) | Accel'n (m/s/s) | vel. (m/s) | T (hrs or days) |
|-----------------|------------|-----------------|------------|-----------------|
| Man - 310 mi | | | | |
| Man - 22 500 mi | | | | |
| Moon | | | | |

9. Explain why the man would want to orbit at 22 500 miles above the surface of the Earth.