## Standing Wave Mathematics <br> Lesson Notes

## Learning Outcomes

- What are the formulas one needs to solve a standing wave problem?
- How does one solve a standing wave problem?


## Standing Waves - Mathematical Relationships

Know: standing wave patterns, relationships, and formulas
Have a strategy!

| Harmonic | Pattern | \# of <br> Nodes | \# of <br> Antinodes | $\lambda$ | f | Examples |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1^{\text {st }}$ |  | 2 | 1 | $\lambda_{1}$ | $\mathrm{f}_{1}$ | 1.20 | 50 |
| $2^{\text {nd }}$ | - | 3 | 2 | $\lambda_{1} / 2$ | $2 \cdot \mathrm{f}_{1}$ | 0.60 | 100 |
| $3^{\text {rd }}$ | 0 | 4 | 3 | $\lambda_{1} / 3$ | $3 \cdot \mathrm{f}_{1}$ | 0.40 | 150 |
| $4^{\text {th }}$ | 0 | 5 | 4 | $\lambda_{1} / 4$ | $4 \cdot \mathrm{f}_{1}$ | 0.30 | 200 |
| $5^{\text {th }}$ | 0 | 6 | 5 | $\lambda_{1} / 5$ | $5 \cdot \mathrm{f}_{1}$ | 0.24 | 250 |
| $\mathrm{n}^{\text {th }}$ | -- | $\mathrm{n}+1$ | n | $\lambda_{1} / n$ | $\mathrm{n} \cdot \mathrm{f}_{1}$ | $1.20 / \mathrm{n}$ | $50 \cdot n$ |

For a string (rope, wire) of length L :

$$
\begin{gathered}
\lambda_{\mathrm{n}}=\lambda_{1} / \mathrm{n} \\
\mathrm{f}_{\mathrm{n}}=\mathrm{n} \cdot \mathrm{f}_{1} \\
\mathrm{v}=\mathrm{f} \cdot \lambda \\
\lambda=(2 / \mathrm{n}) \cdot \mathrm{L} \\
\mathrm{n}=\text { harmonic \# }
\end{gathered}
$$

## An Effective Strategy



## Follow through the video and provide worked-out solutions to the following problems:

Example 1: Solving for $f_{n}$ from $f_{1}$
A string has a fundamental frequency (1st harmonic) of 128 Hz . What is the frequency of the next three harmonics?

Example 2: Solving for $f_{1}$ from $f_{n}$
A string has a frequency of 360 Hz when vibrating as shown.
What is the frequency of the first harmonic?


Example 3: Solving for $\lambda$ or $L$ from $v$ and $f$
A string vibrating with its fourth harmonic frequency of 488 Hz . The wave speed is $366 \mathrm{~m} / \mathrm{s}$ in the string. Determine the length of the string.

Example 4: Solving for $f_{1}$ or $f_{n}$ from $v$ and $L$
Determine the first three harmonic frequencies of a 1.05-m long string in which the wave speed is $198 \mathrm{~m} / \mathrm{s}$.

## Example 5: Solving for v from fand L

A 45.0-cm long string vibrates in its fifth harmonic with a frequency of 220 Hz . Determine the speed at which waves travel through the string.

Example 6: Solving for "Everything"
The string at the right is 6.0 meters long and is vibrating as the third harmonic. The string vibrates up and down with 45 cycles in 10 seconds. Determine the frequency, period, wavelength and speed for this wave.


