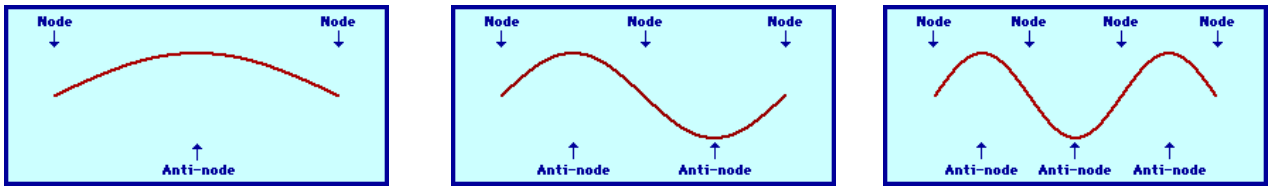
Resonance in Open-End Air Columns
Lesson Notes

Learning Outcomes
- How do you draw the standing wave patterns for the various harmonics of an open-end air column?
- How are the frequencies and wavelengths for the various harmonics related?

Resonance in Strings … Revisited
For string instruments: at just the right frequency, the reflected pulse off a fixed end interferes with incident pulses such that destructive interference (i.e., **nodes**) always occur at the same locations.

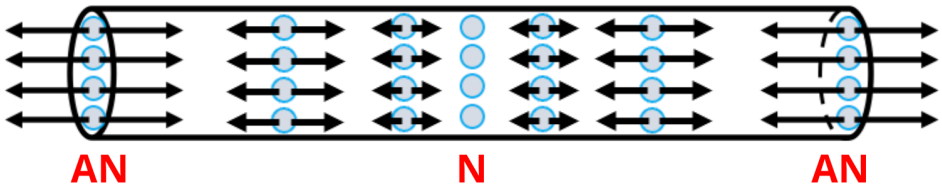
Harmonics
A frequency that results in a standing wave.

1st Harmonic **2nd Harmonic** **3rd Harmonic**

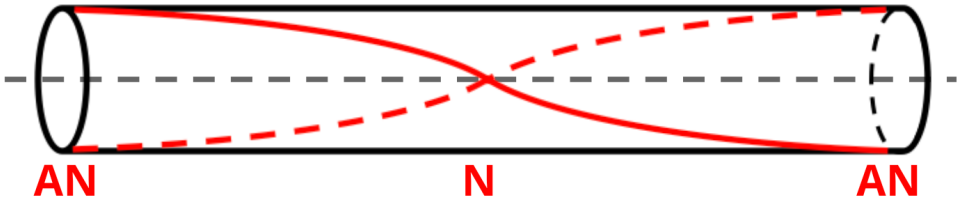


- Nodes are always present at the fixed ends.
- The **fundamental frequency** (or 1st harmonic) is the frequency that results in the longest possible wavelength.

Open-End Air Columns
- The open ends are **anti-nodal positions** (**AN**); air vibrates wildly in and out of the air column.
- **Nodes** (**N**) (where air is undisturbed) are located between anti-nodes.

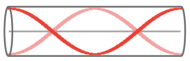
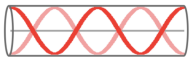


This is the fundamental frequency (1st harmonic):



The above diagram represents a displacement plot.

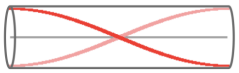
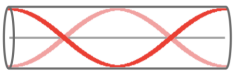
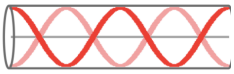
Standing Wave Patterns for Open-End Air Columns

Fundamental (1st H)	2nd Harmonic	3rd Harmonic	4th Harmonic
			
2 AN, 1 N	3 AN, 2 N	4 AN, 3 N	5 AN, 4 N
$L = 0.5 \cdot \lambda_1$	$L = 1.0 \cdot \lambda_2$	$L = 1.5 \cdot \lambda_3$	$L = 2.0 \cdot \lambda_4$
↓ *algebra*	↓ *algebra*	↓ *algebra*	↓ *algebra*
$\lambda_1 = (2/1) \cdot L$	$\lambda_2 = (2/2) \cdot L$	$\lambda_3 = (2/3) \cdot L$	$\lambda_4 = (2/4) \cdot L$

If the air column is 1.2 m long (L=1.2 m), then …

$\lambda_1 = 2.4$ m $\lambda_2 = 1.2$ m $\lambda_3 = 0.8$ m $\lambda_4 = 0.6$ m

Frequency Relationships for Open-End Air Columns

Fundamental (1st H)	2nd Harmonic	3rd Harmonic	4th Harmonic
			
$\lambda_1 = (2/1) \cdot L$	$\lambda_2 = (2/2) \cdot L$	$\lambda_3 = (2/3) \cdot L$	$\lambda_4 = (2/4) \cdot L$

2nd Harmonic: one-half the **λ** of 1st Harmonic … and two times the **f**.

3rd Harmonic: one-third the **λ** of 1st Harmonic … and three times the **f**.

4th Harmonic: one-fourth the **λ** of 1st Harmonic … and four times the **f**.

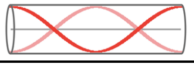
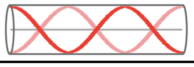
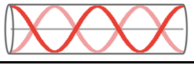
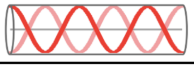
If the fundamental frequency is 250 Hz, then …

$f_1 = 250$ Hz $f_2 = 500$ Hz $f_3 = 750$ Hz $f_4 = 1000$ Hz

Mathematics of Open-End Air Columns

Two general equations for **nth** harmonic:

$\lambda_n = (2/n) \cdot L$ $f_n = n \cdot f_1$

Harmonic	Pattern	# of Nodes	# of Antinodes	λ	f	Examples	
						λ (m)	f (Hz)
1st		1	2	λ_1	f_1	1.20	280
2nd		2	3	$\lambda_1/2$	$2 \cdot f_1$	0.60	560
3rd		3	4	$\lambda_1/3$	$3 \cdot f_1$	0.40	840
4th		4	5	$\lambda_1/4$	$4 \cdot f_1$	0.30	1120
5th		5	6	$\lambda_1/5$	$5 \cdot f_1$	0.24	1400
nth	--	n	n+1	λ_1/n	$n \cdot f_1$	1.20/n	280•n

Based on L = 60 cm

Assume $f_1 = 280$ Hz