Satellite Motion Mathematics
Lesson Notes

Learning Outcomes
- What are the important formulas that describe the speed, acceleration, force, and period for satellites?
- How are the formulas used?

Orbital Speed Derivation
An orbiting satellite having a mass of **M_{sat}** experiences a net force of **$M_{sat} \bullet v^2/R$**. This net force is supplied by gravity and given by the expression **$G \bullet M_{sat} \bullet M_{central}/R^2$**. The **v** is the **orbital speed**, the **R** is the **orbital radius**, and the **$M_{central}$** is the mass of the central body that is being orbited.

$$\cancel{M_{sat}} \bullet \frac{v^2}{\cancel{R}} = G \bullet \frac{\cancel{M_{sat}} \bullet M_{central}}{R^{\cancel{2}}}$$

$$v^2 = \frac{G \bullet M_{central}}{R} \Rightarrow \boxed{v = \sqrt{(G \bullet M_{central}/R)}}$$

Making Sense of Orbital Speed
From the orbital speed equation: $v = \sqrt{(G \bullet M_{central}/R)}$

One would conclude that the speed of an orbiting satellite is independent of the satellite's mass and only dependent upon the radius of orbit (**R**) and the mass of the body being orbited.

- Increase planet mass ⇒ increase orbital speed (**v**).
- Increase orbital radius ⇒ decrease orbital speed (**v**).
- Double planet mass ⇒ increase **v** by a factor of $\sqrt{2}$.
- Triple planet mass ⇒ increase **v** by a factor of $\sqrt{3}$.
- Double radius of orbit ⇒ decrease **v** by a factor of $\sqrt{2}$
- Triple radius of orbit ⇒ decrease **v** by a factor of $\sqrt{3}$.
- Change satellite mass ⇒ No effect upon **v**.

Orbital Acceleration and Force
The acceleration and net force experienced by a satellite is the acceleration caused by gravity and the force of gravity.

$$a = g = G \bullet \frac{M_{central}}{R^2} \qquad F_{net} = F_{grav} = G \bullet \frac{M_{sat} \bullet M_{central}}{R^2}$$

$G = 6.6743 \times 10^{-11}\ N \bullet m^2/kg^2$ R = radius of orbit

Since a satellite moves in a circular path (or close to it), all circular motion equations apply to satellites.

$$v = 2 \bullet \pi \bullet R/T \qquad a = v^2/R \qquad a = 4 \bullet \pi^2 \bullet R/T^2$$

$$F_{net} = m \bullet v^2/R \qquad F_{net} = m \bullet 4 \bullet \pi^2 \bullet R/T^2$$

Orbital Period

The **period** (**T**) is dependent upon the mass of the central body (**$M_{central}$**) and the radius of orbit (**R**): **$T^2 / R^3 = (4 \bullet \pi^2) / (G \bullet M_{central})$**

The above equation can be rearranged to: **$T = \sqrt{[(4 \bullet \pi^2) / (G \bullet M_{central}) \bullet R^3]}$**

Circular motion equations can also be used to calculate the period: **$T = 2 \bullet \pi \bullet R/v$**

Equation Summary

Period (T)	Speed (v)	Acceleration (a)	Net Force (F_{net})
$T = \sqrt{[(4 \bullet \pi^2) / (G \bullet M_{central}) \bullet R^3]}$	$v = \sqrt{(G \bullet M_{central}/R)}$	$a = G \bullet M_{central}/R^2$	$F_{grav} = G \bullet M_c \bullet M_{sat}/R^2$
$T = 2 \bullet \pi \bullet R/v$	$v = 2 \bullet \pi \bullet R/T$	$a = v^2/R$	$F_{net} = m \bullet a$
	$v = \sqrt{a \bullet R}$	$a = F_{net}/m$	$F_{net} = m \bullet v^2/R$

Show solutions to the three Example Problems discussed on Slides 8, 9, and 10:

Example Problem 1

A satellite orbits the earth at a height of 100 km (approximately 60 miles) above the surface. Determine the speed, acceleration and orbital period of the satellite. (Given: **M_{earth}** = 5.97×10^{24} kg, **R_{earth}** = 6.37×10^6 m)

Example Problem 2

The period of the moon is approximately 27.2 days (2.35×10^6 s). Determine the radius of the moon's orbit and the orbital speed of the moon. (Given: **M_{earth}** = 5.97×10^{24} kg)

Example Problem 3

Saturn has many moons. The moon Mimas orbits with an average radius of 1.87×10^8 m. The average orbital period of Mimas is approximately 23 hours (8.28×10^4 s). Determine the mass of Saturn from this information.